Intro Video: Section 4.9
Antiderivatives

Math F251X: Calculus 1

We know how to find a derivative:

$$\text{FUNCTION } f(x) \xrightarrow{d/dx} \text{DERIVATIVE } f'(x)$$

$$F(x) \longleftarrow f(x)$$

$\boxed{\text{An}}$ antiderivative of $f(x)$

$\frac{d}{dx}(x^5) = 5x^4$

$\frac{d}{dx}(\sin(x)) = \cos(x)$

Example: $f(x) = 5x^4 + \cos(x)$

$F(x) = x^5 + \sin(x)$ ← Check: $F'(x) = 5x^4 + \cos(x)$ ✓

$F(x) = x^5 + \sin(x) - 7$ ← check: $F'(x) = 5x^4 + \cos(x)$ ✓

$F(x) = x^5 + \sin(x) - 6{,}735{,}821$ ← check: $F'(x) = 5x^4 + \cos(x)$ ✓

GENERIC: $\boxed{F(x) = x^5 + \sin(x) + C}$ ← C is a constant

What are some antiderivatives we know?

(*particular* antiderivatives)

$\frac{d}{dx}(x^2) = 2x$ antiderivative of x is $\frac{x^2}{2}$

$\frac{d}{dx}(x^3) = 3x^2$ A.D of x^2 is $\frac{x^3}{3}$

if $x \neq 1$, $\frac{d}{dx}(x^n) = nx^{n-1}$ A.D. of x^n is $\frac{x^{n+1}}{n+1}$ $n \neq -1$

$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$ AD of $\frac{1}{x}$ is $\ln|x|$

$\frac{d}{dx}(x) = 1$ AD of 1 is x

$\frac{d}{dx}(e^x) = e^x$ AD of e^x is e^x

More antiderivatives we know

$\frac{d}{dx}(\sin(x)) = \cos(x) \longrightarrow$ AD of $\cos(x)$ is $\sin(x)$

$\frac{d}{dx}(\cos(x)) = -\sin(x) \longrightarrow$ AD of $\sin(x)$ is $-\cos(x)$

$\frac{d}{dx}(\tan(x)) = (\sec(x))^2 \longrightarrow$ AD of $(\sec(x))^2$ is $\tan(x)$

$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x) \longrightarrow$ AD of $\sec(x)\tan(x)$ is $\sec(x)$

$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2} \longrightarrow$ AD of $\frac{1}{1+x^2}$ is $\arctan(x)$

Example: Find any antiderivative of

$$f(x) = 8 - 3x + x^4 - \cos(x)$$

Suppose $F(x)$, $G(x)$ are antiderivatives of $f(x)$, $g(x)$

This says $F'(x) = f(x)$, $G'(x) = g(x)$

Know $\frac{d}{dx}(F(x) + G(x)) = f(x) + g(x)$

$\Rightarrow$ $\boxed{\text{antiderivative of } f(x) + g(x) = F(x) + G(x)}$

If $F(x)$ is an antiderivative of $f(x)$, then

$$\frac{d}{dx}(a F(x)) = a \frac{d}{dx}(F(x)) = a f(x)$$

$\Rightarrow$ $\boxed{\text{antiderivative of } af(x) \text{ is } aF(x).}$

or choose any constant for a particular antiderivative!

Example: $F(x) = 8x - 3 \cdot \frac{x^2}{2} + \frac{x^5}{5} - (\sin(x)) + C$

<u>One more example</u>: A particle travels, and its acceleration is given by the function $a(t) = 2t+1$, and at time $t=0$, its velocity is -2 m/s, and at time $t=1$, its position is at 3. What is the position function $s(t)$?

- $\frac{d}{dt}(\text{velocity}) = \text{acceleration} \Rightarrow$

  velocity = antiderivative of acceleration

  = antiderivative of $2t+1$

  $= 2 \cdot \frac{t^2}{2} + t + c$ Know $v(0) = -2$ $v(t) = t^2 + t - 2$

  So $-2 = 0^2 + 0 + c \Rightarrow \boxed{c = -2}$

- position = antiderivative of velocity = antideriv of $t^2 + t - 2$

  $= \frac{t^3}{3} + \frac{t^2}{2} - 2t + d$

  $s(1) = 3 \Rightarrow 3 = \frac{1}{3} + \frac{1}{2} - 2 + d \Rightarrow d = 5 - \frac{2}{6} - \frac{3}{6} = \frac{25}{6}$ $s(t) = \frac{t^3}{3} + \frac{t^2}{2} - 2t + \frac{25}{6}$